

Nonuniform Spin Triplet Superconductivity due to Antisymmetric Spin-Orbit Coupling in Noncentrosymmetric Superconductor CePt₃Si

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We show that the nonuniform state (Fulde-Ferrel-Larkin-Ovchinnikov (FFLO) state) of the spin triplet superconductivity in noncentrosymmetric systems is stabilized by antisymmetric spin-orbit coupling even if the magnetic field is absent. The transition temperature of the spin triplet superconductivity is reduced by the antisymmetric spin-orbit coupling in general. This pair breaking effect is shown to be similar to the Pauli pair breaking effect due to magnetic field for the spin singlet superconductivity, in which FFLO state is stabilized near the Pauli limit (or Chandrasekhar-Clogston limit) of external magnetic field. Since there are gapless excitations in nonuniform superconducting state, some physical quantities such as specific heat and penetration depth should obey the power low temperature-dependences. We discuss the possibility of the realization of nonuniform state in CePt₃Si.

KEYWORDS: CePt₃Si, heavy fermion superconductivity, spin-orbit coupling, FFLO, spin triplet superconductivity

1. Introduction

Recently, superconductivity in the system without inversion center attracts much interest. The first heavy fermion superconductor without a center of inversion has been observed in CePt₃Si,¹ and several pressure-induced superconductivity in the systems without inversion center have been reported, including CeRhSi₃,⁸ CeIrSi₃⁹ and UIr.¹⁰ In CePt₃Si antiferromagnetic order exists below $T_N = 2.2$ K followed by superconductivity below $T_c = 0.75$ K. The upper critical field $H_{c2}(\sim 5T)$ exceeds the Pauli paramagnetic limiting field (or Chandrasekhar-Clogston limit),¹ which is thought to be an evidence of the spin triplet pairing. Recent experiments on Knight shift^{2,3} also indicate the spin triplet superconductivity.

On the other hand it has been believed that inversion symmetry is required for the spin triplet superconductivity.⁴ Frigeri et al.⁵ have studied the spin triplet superconductivity in the system without inversion symmetry in detail. They took the Rashba type spin-orbit coupling,

$$H_p = \alpha \sum_{\mathbf{k}, s, s'} \mathbf{g}_{\mathbf{k}} \cdot \vec{\sigma}_{s, s'} c_{\mathbf{k}, s}^\dagger c_{\mathbf{k}, s'}, \quad (1)$$

where $c_{\mathbf{k}s}^\dagger$ and $c_{\mathbf{k}s}$ are creation and annihilation operators of electrons with momentum \mathbf{k} and spin s , respectively, and $\mathbf{g}_{\mathbf{k}} = (-k_y, k_x, 0)$ as a model for CePt₃Si, the point group of which is C_{4v} . They obtained that the spin triplet pairing is not affected by this spin-orbit coupling, if the \mathbf{d} -vector of the spin triplet pairing satisfies $\mathbf{d}(\mathbf{k}) \parallel \mathbf{g}_{\mathbf{k}}$, while the transition temperature of the triplet superconductivity is reduced if $\mathbf{d}(\mathbf{k}) \nparallel \mathbf{g}_{\mathbf{k}}$.

Another interesting property of superconductivity in CePt₃Si is the existence of the line nodes in the energy gap indicated by the temperature dependence of penetration depth⁶ and thermal conductivity.⁷ Since the spin

triplet state with $\mathbf{d}(\mathbf{k}) \parallel \mathbf{g}_{\mathbf{k}}$ proposed by Frigeri et al.⁵ has points nodes at $k_x = k_y = 0$ and no line nodes, this state is not consistent with the experiments. In order to explain the existence of the line nodes, the mixing state of triplet and singlet pairings has been studied.^{15,16} The strong spin-orbit coupling and the energy dependence of the density of states (particle-hole asymmetry) have to be assumed to obtain the mixed state of spin singlet and spin triplet.

We propose another possibility, nonuniform spin triplet state, to explain the experiments in CePt₃Si. We will show that there is a similarity between the pair-breaking effect due to the anisotropic spin-orbit coupling for the triplet superconductivity with $\mathbf{d}(\mathbf{k}) \nparallel \mathbf{g}_{\mathbf{k}}$ and the Pauli pair breaking effect due to the magnetic field for the spin singlet superconductivity. In the latter case Fulde and Ferrel,¹¹ and Larkin and Ovchinnikov¹² have shown that the Pauli pair breaking gives rise to a new pairing state between electrons $(\mathbf{k} + \frac{\mathbf{q}}{2}, \uparrow)$ and $(-\mathbf{k} + \frac{\mathbf{q}}{2}, \downarrow)$ on the exchange-split parts of the Fermi surface due to Zeeman effect of magnetic field, if the external magnetic field is close to the Pauli limit. This state is called FFLO state and it has been observed in CeCoIn₅,¹³ recently. In this paper we show that the similar pairing state to FFLO state is possible for the spin triplet superconductivity with $\mathbf{d}(\mathbf{k}) \nparallel \mathbf{g}_{\mathbf{k}}$ even if the magnetic field is absent. In the FFLO state without magnetic field the order parameter is not uniform and the absolute value of the order parameter is zero in parallel planes with period $\frac{2\pi}{|q|}$ in real space. Then some physical properties obey the power-low temperature dependences.

2. Model and order parameters

We adopt a single-band model with electron band energy $\xi_{\mathbf{k}}$ measured relative to the Fermi energy, which is assumed to be spherical when there are no interactions. We use the weak coupling approach, taking the pairing

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interaction as

$$H_V = \frac{1}{2} \sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}} \sum_{s, s'} V_{\mathbf{k}\mathbf{k}'} c_{\mathbf{k}+\frac{\mathbf{q}}{2}, s}^\dagger c_{-\mathbf{k}+\frac{\mathbf{q}}{2}, s'}^\dagger c_{-\mathbf{k}'+\frac{\mathbf{q}}{2}, s'} c_{\mathbf{k}'+\frac{\mathbf{q}}{2}, s}. \quad (2)$$

We assume that the interaction is finite and attractive close to the Fermi energy with the cutoff energy ϵ_c and that the pairing interaction does not depend on the spin and the total momentum of the pair, \mathbf{q} . This Hamiltonian satisfies time reversal and inversion symmetry. The absence of inversion symmetry is introduced by spin-orbit coupling term, H_p (eq. (1)), with

$$\mathbf{g}_{\mathbf{k}} = \sqrt{\frac{3}{2}} \frac{1}{k_F} (-k_y, k_x, 0), \quad (3)$$

as studied by Frigeri et al.⁵ For simplicity, we set $\langle |\mathbf{g}_{\mathbf{k}}|^2 \rangle_{\mathbf{k}} = 1$ where $\langle \cdots \rangle_{\mathbf{k}}$ denotes an average over the Fermi surface. Then, the single-particle Hamiltonian is

$$H_0 = \sum_{\mathbf{k}, s, s'} [\xi_{\mathbf{k}} + \alpha \mathbf{g}_{\mathbf{k}} \cdot \vec{\sigma}]_{ss'} c_{\mathbf{k}s}^\dagger c_{\mathbf{k}s'}. \quad (4)$$

The normal state Green's function is obtained in the 2×2 matrix form as

$$G^0(\mathbf{k}, i\omega_n) = G_+(\mathbf{k}, i\omega_n) \sigma_0 + (\hat{\mathbf{g}}_{\mathbf{k}} \cdot \vec{\sigma}) G_-(\mathbf{k}, i\omega_n). \quad (5)$$

Where

$$G_{\pm}(\mathbf{k}, i\omega_n) = \frac{1}{2} \left[\frac{1}{i\omega_n - \epsilon_{\mathbf{k},+}} \pm \frac{1}{i\omega_n - \epsilon_{\mathbf{k},-}} \right], \quad (6)$$

$$\epsilon_{\mathbf{k},\pm} = \xi_{\mathbf{k}} \pm \alpha |\mathbf{g}_{\mathbf{k}}|, \quad (7)$$

$$\hat{\mathbf{g}}_{\mathbf{k}} = \frac{\mathbf{g}_{\mathbf{k}}}{|\mathbf{g}_{\mathbf{k}}|} = \frac{1}{\sqrt{k_x^2 + k_y^2}} (-k_y, k_x, 0) \quad (8)$$

$\omega_n = (2n+1)\pi k_B T$ is the Matsubara frequency, and σ_0 is the 2×2 unit matrix. When the electrons $(\mathbf{k} + \frac{\mathbf{q}}{2}, s_1)$ and $(-\mathbf{k} + \frac{\mathbf{q}}{2}, s_2)$ make a pair, the linearized gap equation for FFLO state is written in a 2×2 matrix form⁵ as

$$\begin{aligned} \Delta_{ss'}(\mathbf{k} + \frac{\mathbf{q}}{2}) &= -k_B T_c \sum_{\mathbf{k}', n} \sum_{s_1, s_2} V_{\mathbf{k}\mathbf{k}'} G_{ss_1}^0(\mathbf{k}' + \frac{\mathbf{q}}{2}, i\omega_n) \\ &\times \Delta_{s_1 s_2}(\mathbf{k}' + \frac{\mathbf{q}}{2}) G_{s' s_2}^0(-\mathbf{k}' + \frac{\mathbf{q}}{2}, -i\omega_n). \end{aligned} \quad (9)$$

The gap function is decomposed into a spin singlet part $[\psi(\mathbf{k} + \frac{\mathbf{q}}{2})]$ and a spin triplet part $[\mathbf{d}(\mathbf{k} + \frac{\mathbf{q}}{2})]$, as

$$\Delta(\mathbf{k} + \frac{\mathbf{q}}{2}) = [\psi(\mathbf{k} + \frac{\mathbf{q}}{2}) \sigma_0 + \mathbf{d}(\mathbf{k} + \frac{\mathbf{q}}{2}) \cdot \vec{\sigma}] i \sigma_y. \quad (10)$$

Spin triplet part and spin singlet part are mixed in gen-

eral.

$$\begin{aligned} \mathbf{d}(\mathbf{k} + \frac{\mathbf{q}}{2}) &= -k_B T_c \sum_{n, \mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} \\ &\left([G_- G_+ \hat{\mathbf{g}} - G_+ G_- \hat{\mathbf{g}}'] \Psi(\mathbf{k}' + \frac{\mathbf{q}}{2}) \right. \\ &+ G_+ G_+ \mathbf{d}(\mathbf{k}' + \frac{\mathbf{q}}{2}) \\ &- i G_- G_- (\hat{\mathbf{g}} \times \hat{\mathbf{g}}') \Psi(\mathbf{k}' + \frac{\mathbf{q}}{2}) \\ &+ i [G_+ G_- \hat{\mathbf{g}}' + G_- G_+ \hat{\mathbf{g}}] \times \mathbf{d}(\mathbf{k}' + \frac{\mathbf{q}}{2}) \\ &\left. + G_- G_- [\hat{\mathbf{g}} \times \mathbf{d}(\mathbf{k}' + \frac{\mathbf{q}}{2}) \times \hat{\mathbf{g}}' - \hat{\mathbf{g}} \cdot \mathbf{d}(\mathbf{k}' + \frac{\mathbf{q}}{2}) \hat{\mathbf{g}}'] \right), \end{aligned} \quad (11)$$

where we have used the short notation for the products: $G_a G_b = G_a(\mathbf{k} + \frac{\mathbf{q}}{2}, i\omega_n) G_b(-\mathbf{k} + \frac{\mathbf{q}}{2}, -i\omega_n)$ with $a, b = \pm$ and set $\mathbf{g} = \mathbf{g}_{\mathbf{k}'+\frac{\mathbf{q}}{2}}$, $\mathbf{g}' = \mathbf{g}_{-\mathbf{k}'+\frac{\mathbf{q}}{2}}$. We assume that the density of states is constant near the Fermi surface, i.e., the density of states is $N(0)$ even if the Fermi surface is shifted by the spin-orbit coupling. In this assumption, the particle-hole symmetry is satisfied and the singlet and the triplet order parameters do not mix. In other words, $\psi(\mathbf{k}' + \frac{\mathbf{q}}{2})$ terms in the right hand side in eq. (11) vanish. Then we get the gap equation for triplet superconductivity as

$$\begin{aligned} \mathbf{d}(\mathbf{k} + \frac{\mathbf{q}}{2}) &= -k_B T_c \sum_{n, \mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} \left(G_+ G_+ \mathbf{d}(\mathbf{k}' + \frac{\mathbf{q}}{2}) \right. \\ &+ i [G_+ G_- \hat{\mathbf{g}}' + G_- G_+ \hat{\mathbf{g}}] \times \mathbf{d}(\mathbf{k}' + \frac{\mathbf{q}}{2}) \\ &\left. + G_- G_- [\hat{\mathbf{g}} \times \mathbf{d}(\mathbf{k}' + \frac{\mathbf{q}}{2}) \times \hat{\mathbf{g}}' - \hat{\mathbf{g}} \cdot \mathbf{d}(\mathbf{k}' + \frac{\mathbf{q}}{2}) \hat{\mathbf{g}}'] \right). \end{aligned} \quad (12)$$

In the following we set $\mathbf{q} = (0, 0, k_F q)$ and

$$\xi_{\pm \mathbf{k} + \frac{\mathbf{q}}{2}} = \tilde{\xi}_{\mathbf{k}} \pm \frac{\hbar^2 k_F^2 q \cos \theta}{2m}, \quad (13)$$

where $\tilde{\xi}_{\mathbf{k}} = \xi_{\mathbf{k}} + \frac{\hbar^2 k_F^2 q^2}{8m}$. Then we find the transition temperature for spin triplet pairing is given by

$$\begin{aligned} \ln\left(\frac{T_c}{T_{c0}}\right) &= -\pi k_B T_c \langle \text{Im} \sum_{n=0}^{n_c-1} \\ &\left((f_{1-} + f_{2-} + f_{2+} + f_{1+} - \frac{2}{i|\omega_n|}) \left| \mathbf{d}(\mathbf{k} + \frac{\mathbf{q}}{2}) \right|^2 \right. \\ &- (f_{1-} - f_{2-} - f_{2+} + f_{1+}) \\ &\times \left[2(\hat{\mathbf{g}}' \cdot \mathbf{d}(\mathbf{k} + \frac{\mathbf{q}}{2})) (\hat{\mathbf{g}} \cdot \mathbf{d}(\mathbf{k} + \frac{\mathbf{q}}{2})) \right. \\ &\left. \left. - \hat{\mathbf{g}}' \cdot \hat{\mathbf{g}} \left| \mathbf{d}(\mathbf{k} + \frac{\mathbf{q}}{2}) \right|^2 \right] \right) \rangle_{\mathbf{k}}, \end{aligned} \quad (14)$$

where

$$f_{1\pm} = \frac{1}{\frac{\hbar^2 k_F^2 q \cos \theta}{m} + 2i|\omega_n| \pm \alpha(|\mathbf{g}| - |\mathbf{g}'|)}, \quad (15)$$

$$f_{2\pm} = \frac{1}{\frac{\hbar^2 k_F^2 q \cos \theta}{m} + 2i|\omega_n| \pm \alpha(|\mathbf{g}| + |\mathbf{g}'|)}, \quad (16)$$

Basis function	Order parameter
A_1	$\mathbf{d}(\mathbf{k}) = \sqrt{\frac{3}{2}} \frac{1}{k_F} (k_x, k_y, 0)$ $\mathbf{d}(\mathbf{k}) = \frac{1}{k_F} (k_x, k_y, k_z)$ $\mathbf{d}(\mathbf{k}) = \sqrt{\frac{7}{2}} \frac{1}{k_F} (k_x^3, k_y^3, 0)$
A_2	$\mathbf{d}(\mathbf{k}) = \sqrt{\frac{3}{2}} \frac{1}{k_F} (-k_y, k_x, 0)$
B_1	$\mathbf{d}(\mathbf{k}) = \sqrt{\frac{3}{2}} \frac{1}{k_F} (k_x, -k_y, 0)$
B_2	$\mathbf{d}(\mathbf{k}) = \sqrt{\frac{3}{2}} \frac{1}{k_F} (k_y, k_x, 0)$
E	$\mathbf{d}(\mathbf{k}) = \sqrt{3} \frac{1}{k_F} (k_z, 0, 0)$ $\mathbf{d}(\mathbf{k}) = \sqrt{3} \frac{1}{k_F} (0, k_z, 0)$

Table I. Order parameters in C_{4v} point group. The order parameters are normalized as $\langle |\mathbf{d}(\mathbf{k})|^2 \rangle = 1$ for the spherical Fermi surface.

and T_{c0} is the transition temperature for $\alpha = 0$, i.e., $k_B T_{c0} = \epsilon_c \exp(-1/\lambda_t)$ with

$$\lambda_t \mathbf{d}(\mathbf{k}) = -N(0) \langle V_{\mathbf{k}\mathbf{k}'} \mathbf{d}(\mathbf{k}') \rangle_{\mathbf{k}'}. \quad (17)$$

In the following we assume $\mathbf{q} \parallel \hat{z}$, for simplicity. Then $\mathbf{g}_{\mathbf{k}+\frac{\mathbf{q}}{2}}$ is independent of \mathbf{q} , $\mathbf{g}_{\mathbf{k}+\frac{\mathbf{q}}{2}} = \sqrt{\frac{3}{2}} \frac{1}{k_F} (-k_y, k_x, 0)$. In the next section, we calculate the transition temperature as a function of α in $\mathbf{q} = 0$ for the order parameters shown in Table I which are the basis functions for the C_{4v} point group suggested by A. Sergienko and S. H. Curnoe¹⁴ and P. A. Frigeri et al.⁵

3. Transition temperature of the uniform state

Fig. 1 display the transition temperature as a function of α for some types of spin triplet superconductivity with $\bar{q} = 0$. The transition temperature of the state with $\mathbf{d}(\mathbf{k}) \parallel \mathbf{g}_{\mathbf{k}}$ does not depend on α . As shown in Appendix, we obtain that the transition temperature for $\mathbf{d}(\mathbf{k}) = \sqrt{3} \frac{1}{k_F} (k_z, 0, 0)$ and $\mathbf{d}(\mathbf{k}) = \sqrt{3} \frac{1}{k_F} (0, k_z, 0)$ (the two-dimensional representation E) approaches to zero as proportional to $\frac{\pi k_B T_{c0}}{\alpha}$, for large α . The transition temperatures for $\mathbf{d}(\mathbf{k}) = \sqrt{\frac{3}{2}} \frac{1}{k_F} (k_x, -k_y, 0)$ and $\mathbf{d}(\mathbf{k}) = \sqrt{\frac{3}{2}} \frac{1}{k_F} (k_y, k_x, 0)$ also go to zero as proportional to $\frac{\pi k_B T_{c0}}{\alpha}$, for large α . The transition temperature for $\mathbf{d}(\mathbf{k}) = \frac{1}{k_F} (k_x, k_y, k_z)$ is zero at $\alpha \approx 0.31 \pi k_B T_{c0}$ and the transition temperature for $\mathbf{d}(\mathbf{k}) = \sqrt{\frac{3}{2}} \frac{1}{k_F} (k_x, k_y, 0)$ is zero at $\alpha \approx 0.26 \pi k_B T_{c0}$. Interestingly, the transition temperature for $\mathbf{d}(\mathbf{k}) = \sqrt{\frac{7}{2}} \frac{1}{k_F} (k_x^3, k_y^3, 0)$ depends on α in the similar way as the transition temperature for $\mathbf{d}(\mathbf{k}) = \sqrt{\frac{3}{2}} \frac{1}{k_F} (k_x, k_y, 0)$, but it approaches to zero as $(\frac{\alpha}{\pi k_B T_{c0}})^{-19}$ for large α . Curves of T_c vs α for $\mathbf{d}(\mathbf{k}) = \frac{1}{k_F} (k_x, k_y, k_z)$, $\mathbf{d}(\mathbf{k}) = \sqrt{\frac{3}{2}} \frac{1}{k_F} (k_x, k_y, 0)$ and $\mathbf{d}(\mathbf{k}) = \sqrt{\frac{7}{2}} \frac{1}{k_F} (k_x^3, k_y^3, 0)$ are reentrant and they are reminiscent to the transition temperature for the singlet superconductivity as a function of external magnetic field, where only the Pauli pair breaking effects are taken into account and the orbital effects are neglected.^{17,19}

Here we summarize the known results for the Pauli pair breaking effect in the spin singlet superconductivity.^{17,19} In the case of Pauli pair breaking for the spin

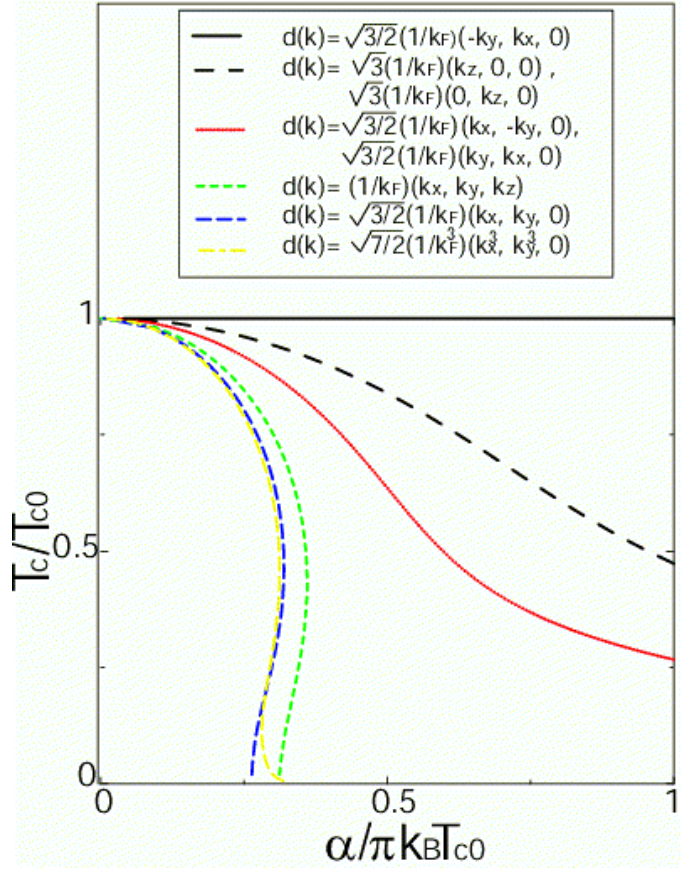


Fig. 1. Transition temperature as a function of the strength of the spin-orbit coupling α for some types of spin triplet superconductivity with $\bar{q} = 0$ and $\mathbf{g}_{\mathbf{k}} = \sqrt{\frac{3}{2}} \frac{1}{k_F} (-k_y, k_x, 0)$. When $\mathbf{d}(\mathbf{k}) \parallel \mathbf{g}_{\mathbf{k}}$, T_c is independent of α . For the states with $\mathbf{d}(\mathbf{k}) \propto (k_z, 0, 0)$, $(0, k_z, 0)$, $(k_x, -k_y, 0)$ and $(k_y, k_x, 0)$, T_c is reduced by α as $T_c \propto 1/\alpha$. For $\mathbf{d}(\mathbf{k}) \propto (k_x, k_y, k_z)$ and $(k_x, k_y, 0)$ T_c becomes zero at the critical value of α . For $\mathbf{d}(\mathbf{k}) \propto (k_x^3, k_y^3, 0)$, T_c is reduced sharply near $\alpha \approx 0.3 \pi k_B T_{c0}$ and $T_c \propto \alpha^{-19}$ at large α .

singlet superconductivity Fermi surfaces is split into two concentric spheres by the external magnetic field. If we assume the second order phase transition into the superconducting state with $\mathbf{q} = 0$, the reentrance is obtained. However, this reentrance cannot be realized, since the first order transition to the superconductivity with $\mathbf{q} = 0$ is expected in the region where the reentrance might exist. The first order transition into the $\mathbf{q} = 0$ superconductivity from the normal state does not realized neither, since it is covered by the second order transition into the pairing with finite \mathbf{q} (FFLO state).

The similar situation is expected in the case of the pair breaking due to an anisotropic spin-orbit coupling. In case of spin-orbit coupling electron energies for the Hamiltonian (eq. (4)) are

$$\epsilon_{\pm} = \xi_{\mathbf{k}} \pm \alpha \sqrt{k_x^2 + k_y^2}, \quad (18)$$

and the eigenstates are

$$\varphi_{\pm} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \mp \frac{k_y - ik_x}{\sqrt{k_y^2 + k_x^2}} \end{pmatrix}, \quad (19)$$

where the upper and the lower components are the up

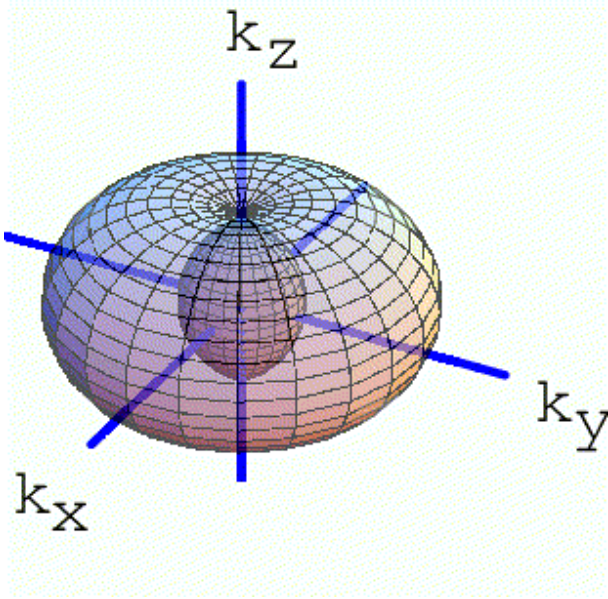


Fig. 2. Fermi surfaces in the presence of the spin-orbit coupling. Spherical Fermi surface ($\xi_{\mathbf{k}} = 0$) is split into two surfaces, $\epsilon_+ = 0$ and $\epsilon_- = 0$.

and down spins. The Fermi surface is split as shown in Fig. 3. In the present case Fermi surfaces is split due to spin-orbit coupling as two spheres with different centers as in Fig. 2 and Fig. 3. The first order transition into the spin triplet state with $\mathbf{q} = 0$ might occur, but we do not consider that possibility in this paper. The second order transition into the state with $\mathbf{q} \neq 0$ in the next section.

4. Nonuniform state of the spin triplet pairing due to anisotropic spin-orbit coupling

In Fig. 4, we plot transition temperatures of the states $\mathbf{d}(\mathbf{k}) = \sqrt{\frac{3}{2}} \frac{1}{k_F} (k_x, k_y, 0)$, as a function of α for some values of \bar{q} , which is defined by

$$\bar{q} = \frac{\hbar^2 k_F^2}{\alpha m} q. \quad (20)$$

For $\bar{q} = 0$, the transition temperature shows a reentrance near $\alpha/(\pi k_B T_{c0}) \approx 0.3$ ($\alpha_c \leq \alpha \leq \alpha_0$, $\alpha_c \approx 0.263\pi k_B T_{c0}$, and $\alpha_0 \approx 0.320\pi k_B T_{c0}$). The reentrance never occurs, since the transition curve is calculated by assuming the second order transition. When the temperature is lowered at fixed α ($\alpha_c \leq \alpha \leq \alpha_0$), the order parameter is finite below the higher transition temperature and the lower transition temperature is spurious, as in the case of the Pauli pair breaking in the spin singlet superconductivity. For the finite value of \bar{q} the critical value α_c at $T_{c0} = 0$ becomes larger. However, the critical value of α never exceed $\alpha_0 \approx 0.32\pi k_B T_{c0}$ for $\bar{q} = 0$. Therefore, contrary to the Pauli pair breaking in the spin singlet pairing, the spin orbit coupling will not stabilize the nonuniform state of $\mathbf{d}(\mathbf{k}) = \sqrt{\frac{3}{2}} \frac{1}{k_F} (k_x, k_y, 0)$.

We obtain the similar α dependence of T_c for the superconducting state of $\mathbf{d}(\mathbf{k}) = \frac{1}{k_F} (k_x, k_y, k_z)$ as shown in Fig. 5. The reentrance seen in the region $0.263 \lesssim \frac{\alpha}{\pi k_B T_{c0}} \lesssim 0.360$ when $\bar{q} = 0$. When $\bar{q} \neq 0$, the reentrant

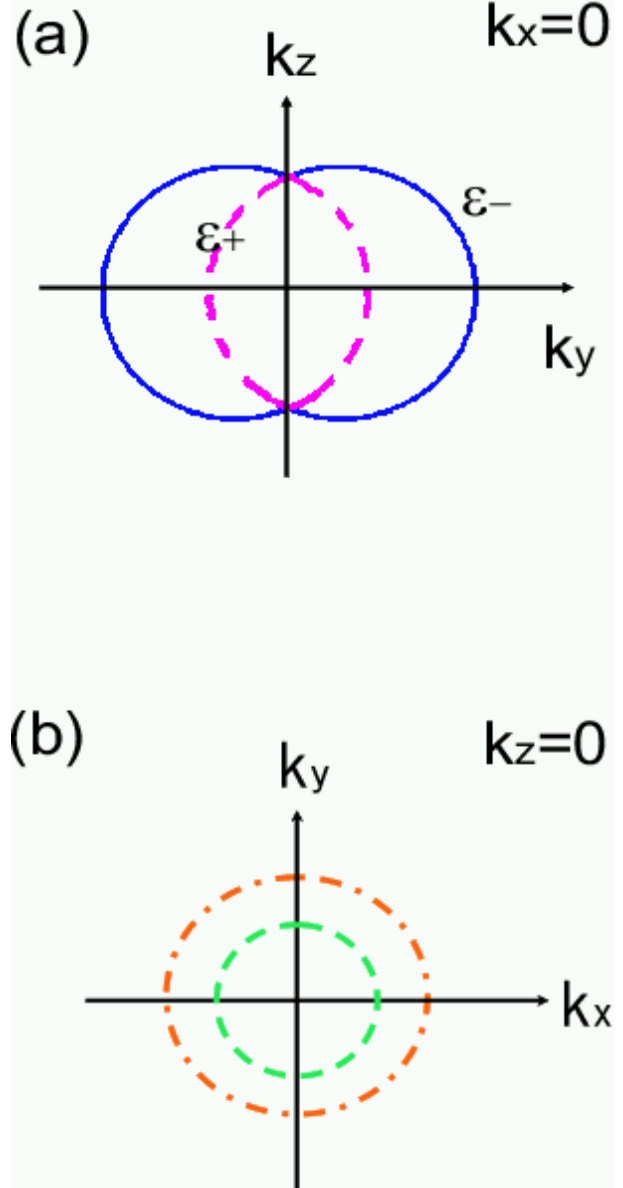


Fig. 3. Cross section of the Fermi surface with the plane $k_x = 0$ (a) and the plane $k_z = 0$ (b).

region becomes small, but the superconducting state is not realized in the region $\alpha \gtrsim 0.360$. Then the nonuniform superconducting state is not stabilized for this order parameter.

For the superconducting state with $\mathbf{d}(\mathbf{k}) = \sqrt{\frac{7}{2}} \frac{1}{k_F^3} (k_x^3, k_y^3, 0)$, we find that the nonuniform state is stabilized in the region $\frac{\alpha}{\pi k_B T_{c0}} \gtrsim 0.312$ as shown in Fig. 6. In this type of the superconductivity T_c is reentrant when $\alpha \lesssim 0.312$ and $T_c \propto \alpha^{-19}$ for $T_c \ll T_{c0}$ and $\alpha \gg \pi k_B T_{c0}$. As seen in Fig. 6, the nonuniform state with $0 < \bar{q} \lesssim 4.5$ is stabilized in the when $\alpha \gtrsim 0.312$.

5. Conclusion

We study the pair breaking effect due to the spin-orbit coupling for the spin triplet superconductivity. We have studied various type of spin triplet superconductivity, which is characterized by the \mathbf{d} vector, the

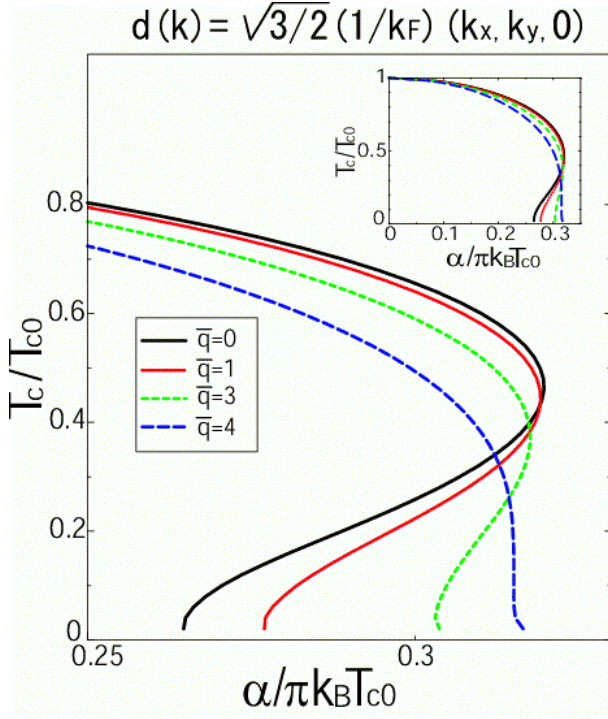


Fig. 4. Transition temperature as a function of α for triplet superconductivity with $\mathbf{g}_{\mathbf{k}} = \sqrt{\frac{3}{2}} \frac{1}{k_F} (-k_y, k_x, 0)$ and $\mathbf{d}(\mathbf{k}) = \sqrt{\frac{3}{2}} \frac{1}{k_F} (k_x, k_y, 0)$.

states with $\mathbf{d}(\mathbf{k}) = \sqrt{\frac{3}{2}} \frac{1}{k_F} (k_x, k_y, 0)$, $\frac{1}{k_F} (k_x, k_y, k_z)$, and $\sqrt{\frac{7}{2}} \frac{1}{k_F} (k_x^3, k_y^3, 0)$ show the reentrant curves of T_c as a function of α . Although the non-uniform state ($\bar{q} \neq 0$, FFLO state) is not stabilized in the former two states, we find that the nonuniform state is stabilized in the latter state. The region of α where the nonuniform state is stabilized is rather small. In this study we have assumed the spherical Fermi surface. If we consider the Fermi surface with different shape with lower dimensionality (quasi-two-dimension or quasi-one dimension) or better nesting condition of the Fermi surface, FFLO state is expected to appear in wider region of parameters, as is the case in the ordinary FFLO state caused by the Pauli pair breaking.¹⁸

If the FFLO state is realized in the absence of magnetic field, the amplitude of the energy gap varies in space and the specific heat and other properties will depend on temperature as a power law. The experimentally observed properties of CePt₃Si, i.e. spin triplet superconductivity with power-law temperature dependences of penetration depth and thermal conductivity, are consistent with the nonuniform spin triplet state due to the spin-orbit coupling.

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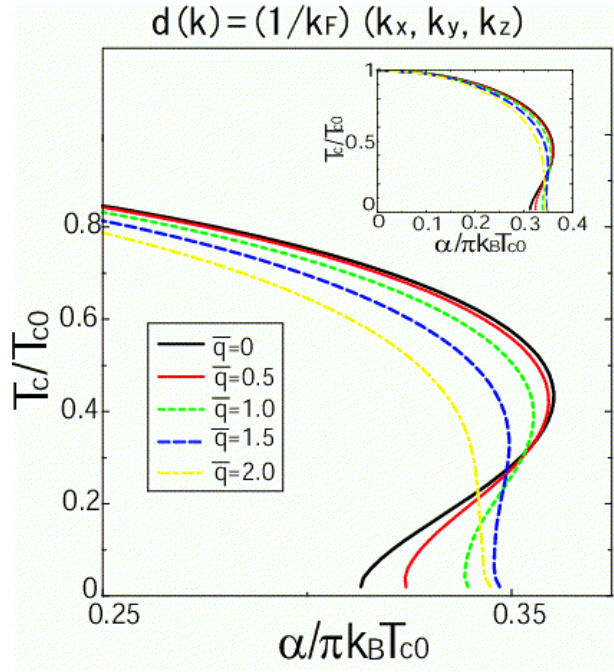


Fig. 5. Transition temperature as a function of α for triplet superconductivity with $\mathbf{g}_{\mathbf{k}} = \sqrt{\frac{3}{2}} \frac{1}{k_F} (-k_y, k_x, 0)$ and $\mathbf{d}(\mathbf{k}) = \frac{1}{k_F} (k_x, k_y, k_z)$.

Appendix: Transition temperature in the case of strong pair breaking due to spin-orbit coupling

In this appendix we calculate the transition temperature of the uniform triplet superconductivity in eq. (14) for $\mathbf{q} = 0$ and $\frac{T_c}{T_{c0}} \rightarrow 0$. For $\mathbf{q} = 0$, eq. (14) becomes

$$\ln\left(\frac{T_c}{T_{c0}}\right) = -2\pi k_B T_c \langle [|\mathbf{d}(\mathbf{k})|^2 - (\hat{\mathbf{g}}_{\mathbf{k}} \cdot \mathbf{d}(\mathbf{k}))^2] \times \text{Im} \sum_{n=0}^{n_c-1} \left(-\frac{1}{i|\omega_n|} + \frac{1}{2i|\omega_n| + 2\alpha|\mathbf{g}_{\mathbf{k}}|} + \frac{1}{2i|\omega_n| - 2\alpha|\mathbf{g}_{\mathbf{k}}|} \right) \rangle_{\mathbf{k}}. \quad (\text{A}\cdot 1)$$

We set $\rho_{\mathbf{k}} = \frac{\alpha|\mathbf{g}_{\mathbf{k}}|}{\pi k_B T_{c0}}$. As we assume that Fermi surface is spherical,

$$k_x = k_F \sin \theta \cos \phi \quad (\text{A}\cdot 2)$$

$$k_y = k_F \sin \theta \sin \phi \quad (\text{A}\cdot 3)$$

$$k_z = k_F \cos \theta \quad (\text{A}\cdot 4)$$

and the average over the Fermi surface is performed as

$$\langle \cdots \rangle_{\mathbf{k}} = \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin \theta \cdots \quad (\text{A}\cdot 5)$$

Then the transition temperature is obtained by

$$\ln\left(\frac{T_c}{T_{c0}}\right) = -\frac{1}{2\pi} \int_0^{2\pi} d\phi \int_{-1}^1 d(\cos \theta) [|\mathbf{d}(\mathbf{k})|^2 - (\hat{\mathbf{g}}_{\mathbf{k}} \cdot \mathbf{d}(\mathbf{k}))^2] \times \frac{1}{2} \text{Re} \left[\Psi\left(n_c + \frac{1}{2}\right) - \Psi\left(\frac{1}{2}\right) - \Psi\left(n_c + \frac{1}{2} + i\frac{\rho_{\mathbf{k}}}{2}\right) + \Psi\left(\frac{1}{2} + i\frac{\rho_{\mathbf{k}}}{2}\right) \right], \quad (\text{A}\cdot 6)$$

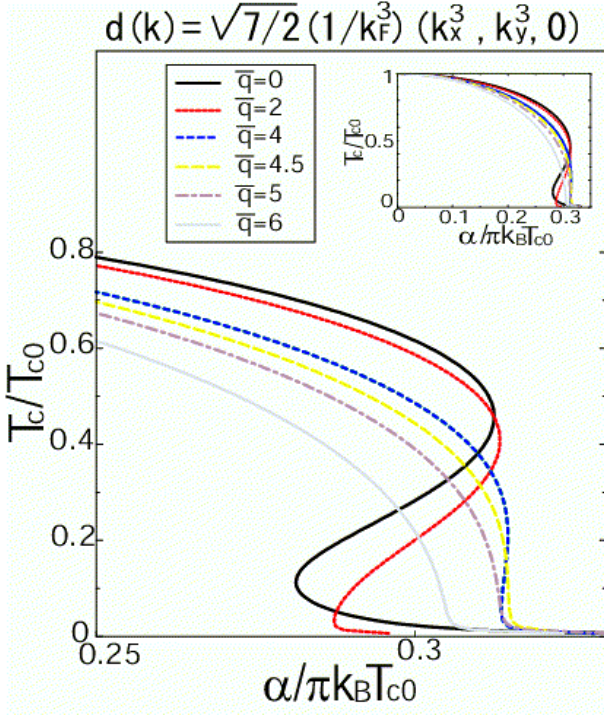


Fig. 6. Transition temperature as a function of α for triplet superconductivity with $\mathbf{g}_{\mathbf{k}} = \sqrt{\frac{3}{2}} \frac{1}{k_F} (-k_y, k_x, 0)$ and $\mathbf{d}(\mathbf{k}) = \sqrt{\frac{7}{2}} \frac{1}{k_F} (k_x^3, k_y^3, 0)$.

where $\Psi(n_c + \frac{1}{2})$ is the digamma function. Since $n_c \gg 1$, we set $\Psi(n_c + \frac{1}{2}) - \Psi(n_c + \frac{1}{2} + i\frac{\rho_{\mathbf{k}}}{2}) \approx 0$ in eq. (A-6). At low temperatures ($T_c \ll T_{c0}$), $\rho_{\mathbf{k}} \gg 1$. Then

$$\begin{aligned} \text{Re} \left(\Psi \left(\frac{1}{2} + i\frac{\rho_{\mathbf{k}}}{2} \right) \right) &\approx \text{Re} \left(\Psi \left(i\frac{\rho_{\mathbf{k}}}{2} \right) \right) \\ &\approx \text{Re} \left(\ln \left(i\frac{\rho_{\mathbf{k}}}{2} \right) \right) \\ &= \ln \left(\frac{\rho_{\mathbf{k}}}{2} \right) \\ &= \ln \left(\frac{\alpha |\mathbf{g}_{\mathbf{k}}|}{2\pi k_B T_c} \right) \end{aligned} \quad (\text{A-7})$$

and the transition temperature in the case of $T_c \ll T_{c0}$ is obtained by

$$\begin{aligned} \ln \left(\frac{T_c}{T_{c0}} \right) &\approx -\frac{1}{4\pi} \int_0^{2\pi} d\phi \int_{-1}^1 d(\cos \theta) [|\mathbf{d}(\mathbf{k})|^2 - (\hat{\mathbf{g}}_{\mathbf{k}} \cdot \mathbf{d}(\mathbf{k}))^2] \\ &\quad \times \ln \left(2e^\gamma \frac{\alpha |\mathbf{g}_{\mathbf{k}}|}{\pi k_B T_c} \right), \end{aligned} \quad (\text{A-8})$$

where we have used $\Psi(\frac{1}{2}) = -\ln(4e^\gamma)$ and γ is the Euler constant.

A.1 $\mathbf{d}(\mathbf{k}) = \sqrt{3} \frac{1}{k_F} (k_z, 0, 0)$

First, we study the order parameter $\mathbf{d}(\mathbf{k}) = \sqrt{3} \frac{1}{k_F} (k_z, 0, 0)$. In this case

$$|\mathbf{d}(\mathbf{k})|^2 - (\hat{\mathbf{g}}_{\mathbf{k}} \cdot \mathbf{d}(\mathbf{k}))^2 = \frac{3}{2} \cos^2 \theta (1 + \cos 2\phi). \quad (\text{A-9})$$

Substituting eq. (A-9) into eq. (A-8), we obtain

$$\begin{aligned} \ln \left(\frac{T_c}{T_{c0}} \right) &= -\frac{1}{4\pi} \int_0^{2\pi} d\phi \int_{-1}^1 dz \frac{3}{2} z^2 (1 + \cos 2\phi) \\ &\quad \times \ln \left(\frac{\sqrt{6(1-z^2)} e^\gamma \alpha}{\pi k_B T_c} \right) \\ &= -\frac{3}{2} \int_0^1 dz \left[z^2 \ln \left(\frac{\sqrt{6} e^\gamma \alpha}{\pi k_B T_c} \right) + \frac{1}{2} z^2 \ln(1-z^2) \right]. \end{aligned} \quad (\text{A-10})$$

We perform the integration over k_z using

$$\int_0^1 dz z^2 \ln(1-z^2) = \frac{2}{9} (\ln 8 - 4), \quad (\text{A-11})$$

and we obtain

$$\ln \left(\frac{T_c}{T_{c0}} \right) = -\frac{1}{2} \left[\ln \left(\frac{\alpha}{\pi k_B T_c} \right) + \ln(2\sqrt{6} e^{\gamma - \frac{4}{3}}) \right]. \quad (\text{A-12})$$

Finally we obtain that the transition temperature for $T_c \ll T_{c0}$ is obtained as

$$\frac{T_c}{T_{c0}} = \frac{1}{2\sqrt{6} e^{\gamma - \frac{4}{3}}} \frac{\pi k_B T_{c0}}{\alpha} \quad (\text{A-13})$$

$$\approx 0.435 \frac{\pi k_B T_{c0}}{\alpha}, \quad (\text{A-14})$$

i.e. T_c is reduced as α^{-1} in the strong spin-orbit coupling case ($\alpha \gg 1$).

A.2 $\mathbf{d}(\mathbf{k}) = \sqrt{\frac{3}{2}} \frac{1}{k_F} (k_x, -k_y, 0)$

In this subsection we study the case of $\mathbf{d}(\mathbf{k}) = \sqrt{\frac{3}{2}} \frac{1}{k_F} (k_x, -k_y, 0)$. In this case we obtain

$$|\mathbf{d}(\mathbf{k})|^2 - (\hat{\mathbf{g}}_{\mathbf{k}} \cdot \mathbf{d}(\mathbf{k}))^2 = \frac{3}{4} (1 - \cos^2 \theta) (1 + \cos 4\phi), \quad (\text{A-15})$$

and we perform the integration over ϕ and k_z in eq. (A-8). We use

$$\int_0^1 dz \ln(1-z^2) = -2 + \ln 4 \quad (\text{A-16})$$

and

$$\int_0^1 dz z^2 \ln(1-z^2) = \frac{9}{2} (\ln 8 - 4). \quad (\text{A-17})$$

In this case we obtain,

$$\begin{aligned} \ln \left(\frac{T_c}{T_{c0}} \right) &= -\frac{1}{4\pi} \cdot \frac{3}{4} \int_0^{2\pi} d\phi \int_{-1}^1 dz (1-z^2) \\ &\quad \times (1 + \cos 4\phi) \ln \left(\frac{\sqrt{6(1-z^2)} e^\gamma \alpha}{\pi k_B T_c} \right) \\ &= -\frac{1}{2} \ln \left(\frac{\sqrt{6} e^\gamma \alpha}{\pi k_B T_c} \right) - \frac{3}{8} (-2 + \ln 4) \\ &\quad + \frac{3}{8} \cdot \frac{2}{9} (3 \ln 2 - \ln e^4). \end{aligned} \quad (\text{A-18})$$

This equation is rewritten as

$$\ln \left(\left(\frac{T_c}{T_{c0}} \right)^2 \left(\frac{\alpha}{\pi k_B T_c} \right) \right) = \ln \left(\frac{e^{\frac{5}{6} - \gamma}}{2\sqrt{6}} \right). \quad (\text{A-19})$$

Finally we obtain

$$\frac{T_c}{T_{c0}} = \frac{1}{2\sqrt{6}e^{\gamma-\frac{5}{6}}} \frac{\pi k_B T_{c0}}{\alpha} \approx 0.263 \frac{\pi k_B T_{c0}}{\alpha}. \quad (\text{A}\cdot 20)$$

A.3 $\mathbf{d}(\mathbf{k}) = \sqrt{\frac{3}{2}} \frac{1}{k_F} (k_y, k_x, 0)$

For $\mathbf{d}(\mathbf{k}) = \sqrt{\frac{3}{2}} \frac{1}{k_F} (k_y, k_x, 0)$ we obtain

$$|\mathbf{d}(\mathbf{k})|^2 - (\hat{\mathbf{g}}_{\mathbf{k}} \cdot \mathbf{d}(\mathbf{k}))^2 = \frac{3}{4} (1 - \cos^2 \theta) (1 - \cos 4\phi). \quad (\text{A}\cdot 21)$$

Since the term proportional to $\cos 4\phi$ vanish in the integration over ϕ , we get the same α dependence of T_c as the case of $\mathbf{d}(\mathbf{k}) = \sqrt{\frac{3}{2}} \frac{1}{k_F} (k_x, -k_y, 0)$.

A.4 $\mathbf{d}(\mathbf{k}) = \frac{1}{k_F} (k_x, k_y, k_z)$

When $\mathbf{d}(\mathbf{k}) = (k_x, k_y, k_z)$, $\hat{\mathbf{g}}_{\mathbf{k}} \cdot \mathbf{d}(\mathbf{k}) = 0$ and

$$|\mathbf{d}(\mathbf{k})|^2 - (\hat{\mathbf{g}}_{\mathbf{k}} \cdot \mathbf{d}(\mathbf{k}))^2 = 1. \quad (\text{A}\cdot 22)$$

We substitute eq. (A·22) into eq. (A·8), and obtain

$$\begin{aligned} \ln\left(\frac{T_c}{T_{c0}}\right) &= -\frac{1}{4\pi} \int_0^{2\pi} d\phi \int_{-1}^1 dz \ln\left(\frac{\sqrt{6(1-z^2)}e^{\gamma}\alpha}{\pi k_B T_c}\right) \\ &= -\ln\left(\frac{\alpha}{\pi k_B T_c}\right) - \ln(2\sqrt{6}e^{\gamma-1}). \end{aligned} \quad (\text{A}\cdot 23)$$

We obtain

$$\frac{\alpha}{\pi k_B T_{c0}} = \frac{1}{2\sqrt{6}e^{\gamma-1}} \approx 0.31 \quad (\text{A}\cdot 24)$$

from eq. (A·23), i.e. $T_c = 0$ at $\alpha \approx 0.31\pi k_B T_{c0}$.

A.5 $\mathbf{d}(\mathbf{k}) = \sqrt{\frac{3}{2}} \frac{1}{k_F} (k_x, k_y, 0)$

When $\mathbf{d}(\mathbf{k}) = \sqrt{\frac{3}{2}} \frac{1}{k_F} (k_x, k_y, 0)$, $\hat{\mathbf{g}}_{\mathbf{k}} \cdot \mathbf{d}(\mathbf{k}) = 0$,

$$|\mathbf{d}(\mathbf{k})|^2 - (\hat{\mathbf{g}}_{\mathbf{k}} \cdot \mathbf{d}(\mathbf{k}))^2 = \frac{3}{2} (1 - \cos^2 \theta). \quad (\text{A}\cdot 25)$$

We substitute this into eq. (A·8),

$$\begin{aligned} \ln\left(\frac{T_c}{T_{c0}}\right) &= -\frac{1}{4\pi} \int_0^{2\pi} d\phi \int_{-1}^1 dz \frac{3}{2} \ln\left(\frac{\sqrt{6(1-z^2)}e^{\gamma}\alpha}{\pi k_B T_c}\right) \\ &= -\ln\left(\frac{\alpha}{\pi k_B T_c}\right) - \ln(2\sqrt{6}e^{\gamma-\frac{5}{6}}). \end{aligned} \quad (\text{A}\cdot 26)$$

Then we obtain

$$\frac{\alpha}{\pi k_B T_{c0}} = \frac{1}{2\sqrt{6}e^{\gamma-\frac{5}{6}}} \approx 0.26, \quad (\text{A}\cdot 27)$$

i.e. $T_c = 0$ at $\alpha \approx 0.26\pi k_B T_{c0}$.

A.6 $\mathbf{d}(\mathbf{k}) = \sqrt{\frac{7}{2}} \frac{1}{k_F^3} (k_x^3, k_y^3, 0)$

When $\mathbf{d}(\mathbf{k}) = \sqrt{\frac{7}{2}} \frac{1}{k_F^3} (k_x^3, k_y^3, 0)$,

$$\begin{aligned} |\mathbf{d}(\mathbf{k})|^2 - (\hat{\mathbf{g}}_{\mathbf{k}} \cdot \mathbf{d}(\mathbf{k}))^2 &= \frac{7}{64} (1 - \cos^2 \theta)^3 (19 + 12 \cos 4\phi + \cos 8\phi). \end{aligned} \quad (\text{A}\cdot 28)$$

Substitute it into eq. (A·8), we obtain

$$\begin{aligned} \ln\left(\frac{T_c}{T_{c0}}\right) &= -\frac{1}{4\pi} \frac{7}{64} \int_0^{2\pi} d\phi \int_{-1}^1 dz (1 - k^2)^3 \\ &\quad \times (19 + 12 \cos 4\phi + \cos 8\phi) \ln\left(\frac{\sqrt{6(1-z^2)}e^{\gamma}\alpha}{\pi k_B T_c}\right). \end{aligned} \quad (\text{A}\cdot 29)$$

Using

$$\int_0^1 dk_z (1 - z^2)^3 \ln(1 - z^2) = -\frac{2552}{3675} + \frac{16}{35} \ln 4, \quad (\text{A}\cdot 30)$$

we obtain from eq. (A·30)

$$\ln\left(\frac{T_c}{T_{c0}}\right) = -\frac{19}{20} \left(\ln\left(\frac{2\sqrt{6}e^{\gamma}\alpha}{\pi k_B T_c}\right)\right) + \frac{19}{16} \cdot \frac{319}{525}, \quad (\text{A}\cdot 31)$$

which is written as

$$\ln\left(\left(\frac{T_c}{T_{c0}}\right)^{\frac{20}{19}} \left(\frac{\alpha}{\pi k_B T_c}\right)\right) = \ln\left(\frac{e^{\frac{319}{400}-\gamma}}{2\sqrt{6}}\right). \quad (\text{A}\cdot 32)$$

Finally we obtain

$$\begin{aligned} \frac{T_c}{T_{c0}} &= \left(\frac{e^{\frac{319}{400}-\gamma}}{2\sqrt{6}}\right)^{19} \left[\frac{\alpha}{\pi k_B T_{c0}}\right]^{-19} \\ &\approx \left(0.254 \frac{\pi k_B T_{c0}}{\alpha}\right)^{19} \\ &\approx 5.08 \times 10^{-12} \left(\frac{\pi k_B T_{c0}}{\alpha}\right)^{19}. \end{aligned} \quad (\text{A}\cdot 33)$$

The transition temperature is reduced as α^{-19} when $\alpha \gg 1$.

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